

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS  
APPLICATIONS  
UNIT 3

**SOLUTIONS**

Section One:  
Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (52 Marks)

This section has**seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (7 marks)

Scatterplots for datasets , and are shown below, together with the least-squares line for each dataset.

<EFOFEX>
id:fxd{7326d55a-ac7e-4181-91b9-c541f4571f39}

FXData:

</EFOFEX>

(a) Which dataset best illustrates the existence of a weak positive relationship between the variables? Justify your choice by referring to two key features of its scatterplot. (3 marks)

|  |
| --- |
| Solution |
| Dataset – the least-squares line for its scatterplot has a positive slope and the points are widely spread around this line indicating a weak relationship. |
| Specific behaviours |
| ✓ correct dataset  ü indicates slope of line is positive  ü indicates points are widely spread around line |

(b) Which dataset best illustrates the existence of a non-linear relationship between the variables. Justify your choice by referring to at least one key feature of its scatterplot.

(2 marks)

|  |
| --- |
| Solution |
| Dataset – the points are not in a straight line but instead closely follow a curve. |
| Specific behaviours |
| ✓ correct dataset  ü indicates points closely follow a curve |

Let and be the values of the correlation coefficient between the variables for datasets , and respectively.

(c) Use estimates for the coefficients to arrange and in ascending numerical order. (2 marks)

|  |
| --- |
| Solution |
| Hence ascending numerical order is . |
| Specific behaviours |
| ✓ indicates reasonable estimates for at least coefficients  ü correct order (full marks if correct without estimations) |

Question 2 (6 marks)

A research institute placed healthy coral in eight reef tanks for a four-week period. For each tank, the average salinity of the water over that time, in parts per thousand, and the percentage of coral that remained healthy at the end of the four weeks was recorded in the table below.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

(a) Construct a scatterplot of the data on the axes below. (2 marks)

<EFOFEX>
id:fxd{af58cd1c-1a47-40b3-b1e1-59a44f049eac}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph. |
| Specific behaviours |
| ✓ correctly plots four points  ü correctly plots all points |

(b) Comment on the validity of each of the following statements:

(i) Average salinity is a good predictor for the percentage of coral that will remain healthy after four weeks in a reef tank. (2 marks)

|  |
| --- |
| Solution |
| Statement is valid, as the scatterplot shows a strong association exists between the variables. |
| Specific behaviours |
| ✓ indicates statement valid  ü reasonable justification of why statement valid |

(ii) Increasing the average salinity causes the percentage of coral that will remain healthy after four weeks in a reef tank to increase. (2 marks)

|  |
| --- |
| Solution |
| Not a valid statement. The observed strong association between the variables does not necessarily mean that there is a causal relationship - the association may be due to a common response to another variable. |
| Specific behaviours |
| ✓ indicates statement not valid  ü reasonable justification of why statement not valid |

Question 3 (7 marks)

Arna has drawn connected graph in the plane to help solve a map colouring problem. Its edges are not directed, it has faces and its adjacency matrix is

(a) Use one or more elements of to explain why

(i) is **not** a complete graph. (2 marks)

|  |
| --- |
| Solution |
| means there is no edge between vertices and , but to be a complete graph every vertex must be directly connected to all other vertices.  [other possible elements (or their mirror image)] |
| Specific behaviours |
| ✓ refers to any zero in the matrix not on the diagonal  ü relates to property of complete graph |

(ii) contains a bridge. (2 marks)

|  |
| --- |
| Solution |
| In the fourth row all elements are zero except for , which means that vertex is only connected to the rest of the graph with one edge, and so this edge is a bridge. |
| Specific behaviours |
| ✓ refers to 4th row or column that contains a single and the rest 0  ü infers that edge must be a bridge |

(b) Show that satisfies Euler’s formula. (3 marks)

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| --- |
| Solution |
| Euler’s formula is , and is given above.  sum of digits above diagonal in matrix .  number of rows (or columns) in matrix .  Hence . |
| Specific behaviours |
| ✓ indicates correct number of edges  ü indicates correct number of vertices  ü correctly uses Euler’s formula |

Question 4 (7 marks)

The first term of a sequence is and the next three terms, in order, are and .

(a) Is this sequence arithmetic, geometric or neither? (1 mark)

|  |
| --- |
| Solution |
| Arithmetic. |
| Specific behaviours |
| ✓ correct answer |

(b) Determine a recurrence relation that defines this sequence. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ states recurrence relation  ü states term of sequence |

(c) Deduce a rule for the term of this sequence. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct rule in form or simplified |

The terms of this sequence model the monthly balance of a savings account. This balance was during the first month.

(d) During which month will this balance first reach or more? (3 marks)

|  |
| --- |
| Solution |
| During the month. |
| Specific behaviours |
| ✓ correctly uses rule from (c) to form equation  ü shows at least one correct simplification step  ü states correct month  *Also accept correct continuation of sequence as justification* |

Question 5 (9 marks)

(a) The following divided bar graph shows the writing preferences (pencil or pen) for a sample of left and right-handed students.

<EFOFEX>
id:fxd{7ecbbccc-614f-4f95-8a0f-35ba753f2e08}

FXData:

</EFOFEX>

Explain whether the data indicates the presence of an association between dominant hand and writing preference. (2 marks)

|  |
| --- |
| Solution |
| Yes, the graph suggests an association between the variables exists since approximately (28) of left-handed students prefer pencil compared to approximately 5 (48) of right-handed students – a significant difference.  (or for pens ~70% (72) LH and ~50% (52) |
| Specific behaviours |
| ✓ indicates association  ü explanation using different percentages for either pencil or pen |

(b) The table below shows the responses of people who house share in either Sydney or Adelaide to a survey about their financial position pre-COVID to now.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Financial position pre-COVID to now | | |
| Better | Same | Worse |
| Sydney |  |  |  |
| Adelaide |  |  |  |

(i) Calculate how many more people responded to the survey in Sydney compared to Adelaide. (2 marks)

|  |
| --- |
| Solution |
| .  Hence more people. |
| Specific behaviours |
| ✓ one correct city total  ü correct difference |

(ii) State the explanatory variable for this data. (1 mark)

|  |
| --- |
| Solution |
| People who house share in Sydney or Adelaide. |
| Specific behaviours |
| ✓ correct answer |

(iii) Complete the appropriate percentaged table for the data. (2 marks)

|  |
| --- |
| Solution |
| Sydney figures are divided by to obtain percentages.  Adelaide figures are doubled to obtain percentages.   |  |  |  |  | | --- | --- | --- | --- | |  | Financial position pre-COVID to now | | | | Better | Same | Worse | | Sydney |  |  |  | | Adelaide |  |  |  | |
| Specific behaviours |
| ü each row adds to (row percentages calculated)  ü correct table |

(iv) State, with reasons, whether the data suggests an association is present between the two variables. (2 marks)

|  |
| --- |
| Solution |
| The data does not suggest an association exists between the variables as no significant difference is observed between each pair of percentages in any of the three columns. |
| Specific behaviours |
| ✓ indicates no evidence of an association  ü explanation using similar percentages comparing down a column |

Question 6 (10 marks)

Mack delivers milk from his depot at to all grocery shops located at and daily.

In the graph below the vertices represent the depot and shops, the edges represent direct roads between shops and the edge weights are the travel times along these roads in minutes.

<EFOFEX>
id:fxd{2e8a484a-8982-4180-abea-46245737407e}

FXData:

</EFOFEX>

(a) Explain why the graph shown above is Hamiltonian. (2 marks)

|  |
| --- |
| Solution |
| The graph contains a cycle (closed path) that includes all vertices once. |
| Specific behaviours |
| ✓ states cycle or closed path  ü states cycle includes all vertices once |

Mack always starts and finishes a daily delivery from his depot, and never visits a shop or travels along the same road more than once. He always spends minutes at each shop to complete the delivery.

(b) One day, Mack left the depot at am and started his delivery round by travelling to shop , as it had requested an urgent delivery. List the order that the shops received their deliveries and determine the time that Mack returned to the depot on this day. (4 marks)

|  |
| --- |
| Solution |
| Delivery order .  Journey time:  Time at shops:  Returns at am. |
| Specific behaviours |
| ✓ correct order of deliveries (doesn’t have to show start & finish at C)  ü indicates correct journey time  ü correctly allows for time at each shop  ü correct return time |

(c) Determine the order in which Mack should visit the shops so that he returns to the depot in the least possible time, and state the length of time this is. Justify your answer.

A copy of the graph has been provided below. (4 marks)

<EFOFEX>
id:fxd{2e8a484a-8982-4180-abea-46245737407e}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| Order for shortest Hamilton cycle is:  Least possible time: minutes.  Other possible Hamilton cycles with times are:  *NB All orders listed above may be reversed* |
| Specific behaviours |
| ✓ lists two or more alternative Hamilton cycles to part (b)  ü correctly calculates one alternative journey time  ü identifies shortest possible Hamilton cycle  ü correctly states least possible journey time |

Question 7 (6 marks)

For each of the following questions, answer yes or no and then justify your answer.

(a) Is the graph shown below bipartite? (2 marks)

|  |
| --- |
| Solution |
| Yes – the vertices can be split into two distinct groups:  <EFOFEX> id:fxd{059ea6b8-d0dc-4e6c-adf8-06c836d3e157}  FXData:  </EFOFEX> |
| Specific behaviours |
| ✓ answers yes  ü any correct justification |

<EFOFEX>
id:fxd{d3d76542-47a7-481b-9ed1-15e7a18bfc19}

FXData:

</EFOFEX>

(b) Is a connected planar graph with vertices and edges a simple graph? (2 marks)

|  |
| --- |
| Solution |
| No. A complete graph with vertices (, shown below) has edges, and so the seventh must be either a loop or multiple edge and a simple graph does not contain these.  <EFOFEX> id:fxd{70f1c9fb-47d9-43a4-9771-6270c8a372c0}  FXData:  </EFOFEX> |
| Specific behaviours |
| ✓ answers no  ü justifies using properties of simple graph |

(c) Is graph a subgraph of graph ? (2 marks)

<EFOFEX>
id:fxd{aea4f9b6-b702-4f4a-984d-4cab82fffa9b}

FXData:

</EFOFEX>

**End of questions**

|  |
| --- |
| Solution |
| No. The edge between vertices and in is not an edge in . |
| Specific behaviours |
| ✓ answers no  ü any correct justification |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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